17216

11819

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Solve any TEN of the following: **20**

a) State whether the function is even or odd

if $f(x) = \frac{a^x + a^{-x}}{2}$

b) If $f(x) = x^2 \frac{1}{x^2}$

show that, $f(x) + f(-x) = 2 f(x)$

c) Seperate into real and immaginary part for, sin $(x + iy)$

d) If $(3 + i)x + (1 - i)y = 1 + 7i$, find x and y.

e) Evaluate $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

f) Evaluate $\lim_{x \to 0} \left[\frac{5 \tan x + 6x}{9x - 2 \sin x} \right]$

P.T.O.

Marks

g) Evaluate $\lim_{x \to 0} \frac{a^x - b^x}{x}$

h) If $y = x \cdot \log x$ find $\frac{dy}{dx}$

i) If $y = \cos(\log x)$ find $\frac{dy}{dx}$

j) Differentiate $\sin x$ w.r.t. 'log x'

k) From the following system of Equations,

$3x + 2y = 4.5,\ 2x + 3y - z = 5,\ -y + 2z = 0.52$

Find one iteration only using Gauss-Seidal method.

l) Show that the root of the equation $x^3 - 9x + 1 = 0$ lies betⁿ 2 and 3.

2. Solve any <u>FOUR</u> of the following: **16**

a) Express $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ in polar form.

b) Simplify using De Moivre's theorem

$$\frac{\left[\cos\frac{4}{3}\theta + i\sin\frac{4}{3}\theta\right]^3 \cdot \left[\cos\frac{1}{2}\theta - i\sin\frac{1}{2}\theta\right]^2}{\left[\cos 4\theta - i\sin 4\theta\right] \cdot \left[\cos 2\theta + i\sin 2\theta\right]^3}$$

c) By using De Moivre's theorem find "cube root of unity".

d) Show that $\sin 2\theta = 2\sin\theta \cdot \cos\theta$ using Euler's form.

e) If $f(x) = 50\sin[100\pi x + 0.4]$, prove that $f\left[\frac{1}{50} + x\right] = f(x)$

f) If $f(x) = \frac{x+3}{4x-5}$ and $t = \frac{3+5x}{4x-1}$

show that $f(t) = x$

3. Solve any <u>FOUR</u> of the following: **16**

a) If $f(x) = x^2 + 5$, find x if $f(x+2) = f(x-2)$

b) If $f(x) = 16^x + \log_2 x$ find $f\left(\frac{1}{4}\right)$

Marks

c) Evaluate $\lim_{x \to 0} \frac{\sin 2x^0}{x}$

d) Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$

e) Evaluate $\lim_{x \to 0} \frac{12^x - 4^x - 3^x + 1}{x^2}$

f) Evaluate $\lim_{x \to \infty} \left[\frac{x+1}{x+2}\right]^x$

4. Solve any FOUR of the following: **16**

a) Using 1st principle of derivatives find derivatives of $f(x) = \log x$

b) If u and v are differentiable functions of x, then prove that $\frac{d}{dx}\left[\text{U.V}\right] = u\frac{dv}{dx} + v\frac{du}{dx}$

c) Find $\frac{dy}{dx}$ *if* $y = x^x + a^x$

d) Find $\frac{dy}{dx}$ *if* $y = \log\left[x^2 - 2x + \sin x\right]$

e) Find $\frac{dy}{dx}$ *if* $y = \tan^{-1}\left[x \Big/ \left(1 + 12x^2\right)\right]$

f) Find $\frac{dy}{dx}$ *if* $x^2 + y^2 = 4xy$

5. Solve any FOUR of the following: **16**

a) Evaluate $\lim_{x \to 0} \frac{\log(a+x) - \log a}{x}$

b) Evaluate $\lim_{x \to 0} \frac{3\sin x - \sin 3x}{x^3}$

c) Using bisection method find approximate root of the equation $x^3 - 4x - 9 = 0$

d) Find approximate root of equation, $x \cdot \log_e x = 1.2$ by using bisection method.

e) Find root of the equation $x^2 + x - 3 = 0$ using Regula Falsi method.

Marks

f) Use Newton-Raphson method to find root of equation $x^2 + x - 3 = 0$ (upto three iterations)

6. Solve any <u>FOUR</u> of the following: **16**

a) If $y = 2\cos[\log x] + 3\sin[\log x]$ prove that $x^2 \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} + y = 0$

b) If $x = a[\theta - \sin\theta]$ and $y = a[1 - \cos\theta]$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $\theta = \pi/4$

c) Solve the following equations by Gauss elimination method.

$4x + y + 2z = 12,\ -x + 11y + 4z = 33,\ 2x - 3y + 8z = 20$

d) Solve the following equation by using Jacobi's method,

$20x + y - 2z = 17,\ 3x + 20y - z + 18 = 0,\ 2x - 3y + 20z = 25$

e) Solve the following equation by using Gauss elimination method.

$2x + 3y + z = 13,\ x - y - 2z + 1 = 0,\ 3x + y + 4z = 15$

f) Solve the following equation using Gauss Seidal method.

$10x + y + z = 12,\ x + 10y + z = 12,\ x + y + 10z = 12$